

**Math 101 – WORKSHEET 17**  
**IMPROPER INTEGRALS**

1. IMPROPER AT INFINITY

- (1) For which values of  $p$  does  $\int_1^{\infty} \frac{1}{x^p} dx$  converge? Diverge?
- (2) (Final, 2010) Evaluate  $\int_{-\infty}^{-1} e^{2x} dx$ . Simplify your answer as much as possible.
- (3) Find a constant  $C$  such that  $\int_{-\infty}^{+\infty} \frac{C dx}{1+x^2} = 1$ .
- (4) We study  $\int_{-\infty}^{+\infty} x dx$ .
- (a) Evaluate  $\int_{-T}^T x dx$ .
  - (b) Evaluate  $\lim_{T \rightarrow \infty} \int_{-T}^T x dx$ .
  - (c) Does the integral converge?
- (5) (Final, 2009) For what values of  $p$  does  $\int_e^{\infty} \frac{dx}{x(\log x)^p}$  converge?

## 2. IMPROPER AT FINITE POINTS

(6) For which values of  $p$  does  $\int_0^1 \frac{dx}{x^p}$  converge?

(7) (Math 103 Final, 2013) Evaluate the integral if it exists, otherwise show that it doesn't:  $I = \int_0^2 \frac{dx}{1-x^2}$ .

## 3. COMPARISON OF INTEGRALS

(8) Decide which of the following integrals converge

(a) (103 Final, 2012)  $\int_1^\infty \frac{1+\sin x}{x^2} dx$ .

(b)  $\int_1^\infty \frac{3-\cos x}{x} dx$ .

(c) (Bell curve)  $\int_{-\infty}^{+\infty} e^{-x^2} dx$

(d)  $\int_0^1 \frac{dx}{\sqrt{x+\sin x}}$

(e) (hard)  $\int_0^1 \frac{dx}{x^2+x^3}$

(f) (hard)  $\int_0^\infty \frac{x^{1000}}{e^x} dx$