Math 101 – WORKSHEET 15
PARTIAL FRACTIONS, APPROXIMATE INTEGRATION

1. Partial fractions expansion

Theorem. *Let $\frac{p}{q}$ be a rational root of $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ where $a_i \in \mathbb{Z}$. Then $p|a_0$ and $q|a_n$.*

(1) Write the form of the partial fraction expansion: $\frac{x^2+3x+6}{x^3(2x+3)(x-1)^2} =$

(2) (Division)
  (a) Factor $x^3 - 6x^2 + 11x - 6$

  (b) Divide with remainder: $\frac{2x^3}{(x+2)(2x+3)} =$

(3) Consider $\frac{8x-10}{4x^3-4x^2+5x} = \frac{8x-10}{x(4x^2-4x+5)} = \frac{A}{x} + \frac{Bx+C}{4x^2-4x+5}$
  (a) Find A using method 2

  (b) Calculate $\frac{8x-10}{x(4x^2-4x+5)} - \frac{A}{x}$ to find B, C.

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2. Numerical integration

To approximate $I = \int_a^b f(x)\,\mathrm{d}x$ using n intervals:

- $\Delta x = \frac{b-a}{n}$. $x_i = a + i\Delta x = a + \frac{i}{n}(b-a)$. For midpoint rule, $\bar{x}_i = x_i - \frac{1}{2}\Delta x = a + \left(i - \frac{1}{2}\right)\Delta x$.
- Midpoint rule: $\int_a^b f(x)\,\mathrm{d}x \approx \Delta x\,(f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n))$.
- Trapezoid rule: $\int_a^b f(x)\,\mathrm{d}x \approx \frac{\Delta x}{2}\,(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$.
- Simpson's rule (n even)
  $$\int_a^b f(x)\,\mathrm{d}x \approx \frac{\Delta x}{3}\,(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)).$$

You must memorize these

(1) (Final 2009) Let $f(x) = \sin(e^x)$. Approximate $I = \int_0^2 f(x)\,\mathrm{d}x$ using the modpoint rule, the trapezoid rule, and Simpson's rule, with $n = 4$ in all cases. You may leave your answers in calculator-ready form.

(2) (Final 2015) Write down the Simpson's rule approximation to $I = \int_0^2 (x-3)^5\,\mathrm{d}x$ with $n = 6$.You may leave your answers in calculator-ready form.