

Math 101 – WORKSHEET 11
TRIGONOMETRIC INTEGRALS

Formulas to memorize: $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$

$$\sin(2x) = 2 \sin x \cos x \quad \cos(2x) = 2 \cos^2 x - 1 \quad \cos^2 x = \frac{1 + \cos(2x)}{2} \quad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

- (1) Evaluate the integrals

(a) $\int \sin^4 x \cos^3 x \, dx$

(b) $\int \sin^5 x \cos^4 x \, dx$

(c) $\int \sin^4 x \cos^4 x \, dx$

(d) $\int \sin^5 x \cos^3 x \, dx$

Memorize: $\boxed{\sec x = \frac{1}{\cos x}, \quad (\tan x)' = \sec^2 x = 1 + \tan^2 x, \quad (\sec x)' = \sec x \tan x}.$

(2) Powers of tangent and secant

(a) Evaluate $\int_0^{\pi/4} \tan x \, dx$

(b) Evaluate $\int_{-\pi/4}^{+\pi/4} \tan x \, dx$

(c) (even power of secant) Evaluate $\int \tan^5 x \sec^4 x \, dx$ using the substitution $u = \tan x$.

(d) (odd power of tangent) Write $\int \tan^5 x \sec^3 x \, dx$ in the form $\int \sin^n x \cos^m x \, dx$ and evaluate it.