Math 101 – SOLUTIONS TO WORKSHEET 2
AREA UNDER A CURVE

(1) Let A be the area lying between the x-axis, the curve $y = x^2$ and the lines $x = 0$, $x = 1$.

    (a) Draw a picture

        Solution: TBA

    (b) Dividing the interval $[0, 1]$ into two equal-width strips, show that $A \leq \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{2} \cdot 1^2 = \frac{5}{8}$.

    (c) Using the same subdivision, show that $A \geq \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{8}$.

        Solution: TBA

    (d) Using a subdivision into 3 strips, show $\frac{1}{3} \cdot 0^2 + \frac{1}{3} \left(\frac{1}{3}\right)^2 + \frac{1}{3} \left(\frac{2}{3}\right)^2 \leq A \leq \frac{1}{3} \left(\frac{1}{3}\right)^2 + \frac{1}{3} \left(\frac{2}{3}\right)^2 + \frac{1}{3} \cdot 1^2$.

        Solution: TBA

    (e) For better accuracy, we use rectangles whose height is given by the function value at the *middle* of the strip. What do you get now?

      Solution: TBA