Math 101 – WORKSHEET 19
IMPROPER INTEGRALS

1. IMPROPER AT INFINITY

(1) For which values of p does $\int_1^\infty \frac{1}{x^p}\,\mathrm{d}x$ converge? Diverge?

(2) (Final, 2010) Evaluate $\int_{-\infty}^{-1} e^{2x}\,\mathrm{d}x$. Simplify your answer as much as possible.

(3) Find a constant C such that $\int_{-\infty}^{+\infty} \frac{C\,\mathrm{d}x}{1+x^2} = 1$.

(4) We study $\int_{-\infty}^{+\infty} x\,\mathrm{d}x$.
   (a) Evaluate $\int_{-T}^{T} x\,\mathrm{d}x$.
   (b) Evaluate $\lim_{T\to\infty} \int_{-T}^{T} x\,\mathrm{d}x$.
   (c) Does the integral converge?

(5) (Final, 2009) For what values of p does $\int_e^\infty \frac{\mathrm{d}x}{x(\log x)^p}$ converge?

2. Improper at finite points

(6) For which values of p does $\int_0^1 \frac{\mathrm{d}x}{x^p}$ converge?

(7) (Math 103 Final, 2013) Evaluate the integral if it exists, otherwise show that it doesn't: $I = \int_0^2 \frac{\mathrm{d}x}{1-x^2}$.

3. Comparison of integrals

(7) Decide which of the following integrals converge
    (a) (103 Final, 2012) $\int_1^\infty \frac{1+\sin x}{x^2}\, \mathrm{d}x$.

    (b) $\int_1^\infty \frac{3-\cos x}{x}\, \mathrm{d}x$.

    (c) (Bell curve) $\int_{-\infty}^{+\infty} e^{-x^2}\, \mathrm{d}x$

    (d) $\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}+\sin x}$

    (e) (hard) $\int_0^1 \frac{\mathrm{d}x}{x^2+x^3}$

    (f) (hard) $\int_0^\infty \frac{x^{1000}}{e^x}\, \mathrm{d}x$