Math 101 – SOLUTIONS TO WORKSHEET 16
PARTIAL FRACTIONS, APPROXIMATE INTEGRATION

1. Partial fractions expansion

(1) Apply Method 2 to find A, B, C such that
$\frac{6x^2-26x+26}{x^3-6x^2+11x-6} = \frac{6x^2-26x+26}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$
Solution: We have

$$\frac{6x^2-26x+26}{(x-1)(x-2)(x-3)} \sim_1 \frac{6\cdot 1^2 - 26\cdot 1 + 26}{(x-1)(1-2)(1-3)} = \frac{6}{(x-1)(-1)(-2)} = \frac{3}{x-1}$$
$$\frac{6x^2-26x+26}{(x-1)(x-2)(x-3)} \sim_2 \frac{6\cdot 2^2 - 26\cdot 2 + 26}{(2-1)(x-2)(2-3)} = \frac{-2}{(-1)(x-2)} = \frac{2}{x-1}$$
$$\frac{6x^2-26x+26}{(x-1)(x-2)(x-3)} \sim_3 \frac{6\cdot 3^2 - 26\cdot 3 + 26}{(3-1)(3-2)(x-3)} = \frac{2}{(2)(x-2)} = \frac{1}{x-1}$$

so $A = 3$, $B = 2$, $C = 1$.

(2) Now consider $\frac{8x-10}{4x^3-4x^2+5x} = \frac{8x-10}{x(4x^2-4x+5)} = \frac{A}{x} + \frac{Bx+C}{4x^2-4x+5}$

(a) Find A using method 2
Solution: We have $\frac{8x-10}{x(4x^2-4x+5)} \sim_0 \frac{-10}{x(5)} = \frac{-2}{x}$ so $A = -2$.

(b) Calculate $\frac{8x-10}{x(4x^2-4x+5)} - \frac{A}{x}$ to find B, C.
Solution: We have

$$\begin{aligned}
\frac{8x-10}{x(4x^2-4x+5)} - \frac{(-2)}{x} &= \frac{1}{x}\left[\frac{8x-10}{4x^2-4x+5} + 2\right] \\
&= \frac{1}{x}\left[\frac{8x-10+2\left(4x^2-4x+5\right)}{4x^2-4x+5}\right] \\
&= \frac{1}{x}\left[\frac{8x^2+8x-8x-10+10}{4x^2-4x+5}\right] \\
&= \frac{8x^2}{x(4x^2-4x+5)} \\
&= \frac{8x}{4x^2-4x+5}
\end{aligned}$$

so that $B = 8$ and $C = 0$.

(3) Finally consider $\frac{x^2}{(x+2)(2x-3)}$. Can we have A, B such that $x^2 = A(x+2) + B(2x-3)$?
Solution: No, because the degrees don't match.

Date: 10/2/2016, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Approximate integration

Let $f(x) = \sin(x^2)$. Estimate $\int_0^1 f(x)\,\mathrm{d}x$ using the trapezoid rule, the midpoint rule, and Simpson's rule, with $n = 4$ in all cases. You may leave your answers in calculator-ready form.