Math 101 – WORKSHEET Special
AVERAGE VALUE

1. Average Value

In this note I collect a few examples of computing the average value of a function, and some example problems using it.

Definition. Let f be defined and integrable on $[a, b]$. The *average value* of f on the interval is

$$\boxed{\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x}.$$

Remark 1. A Riemann sum for $\int_a^b f\,\mathrm{d}x$ is $\sum_{i=1}^n f(x_i^*)\Delta x = \sum_{i=1}^n f(x_i^*)\frac{b-a}{n}$; dividing by $b-a$ we see that a Riemann sum of the integral above is:

$$\frac{1}{n}\sum_{i=1}^n f(x_i^*)\,.$$

In other words, the average value of f on the interval is the limit of *averages of values of f at sample points.*

In straightforward problems you are given f, a, b and asked to compute the average. In more complicated problems a, b or f itself may depend on a parameter, and you need to have the confidence to compute the average in terms of the parameter, geting a formula instead of a numerical answer for the average value. You can then *solve* for the parameter using given information.

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2. Straight-up problems

In these problems, simply compute the average value of the given function on the given interval.

(1) $f(x) = e^{5x} + x\sqrt{x^2+1}$ on the interval $[-1, 2]$.

(2) (Final, 2009) $f(\theta) = |\sin\theta - \cos\theta|$ on $\left[0, \frac{\pi}{2}\right]$.

(3) (Final, 2011) $f(x) = xe^x$ on $[0, 2]$.

3. Problems involving a parameter

In the following problems, one piece of information (the function f or the interval) depends on a parameter. You need to compute the average value using the parameter, and then solve for the parameter.

(1) (Final, 2012) Let k be a positive constant. Find the average value of $f(x) = \sin(kx)$ on $[0, \pi/k]$.

(2) Let $f(x) = x\sqrt{x^2 + r^2}$. For what value of $r > 0$ is the average value of f on $[0, 3]$ equal to $\frac{1}{9}$?

(3) (Final, 2010) Find a number $b > 0$ such that the function $f(x) = x - 1$ has average value 0 on the interval $[0, b]$.