

**MATH 100 – WORKSHEET 18**  
**THE MVT AND CURVE SKETCHING**

1. THE MEAN VALUE THEOREM

**Theorem.** *Let  $f$  be defined and differentiable on  $[a, b]$ . Then there is  $c$  between  $a, b$  such that  $\frac{f(b)-f(a)}{b-a} = f'(c)$ . Equivalently, for any  $x$  there is  $c$  between  $a, x$  so that  $f(x) = f(a) + f'(c)(x - a)$ .*

(1) Suppose  $f(1) = 3$  and  $-3 \leq f'(x) \leq 2$  for  $x \in [1, 4]$ . What can you say about  $f(4)$ ?

(2) Suppose  $f'(x) = \frac{e^x}{x+\pi}$  for  $0 \leq x \leq 2$ . How large can  $f(2) - f(0)$  be?

(3) Suppose  $f'(x) > 0$  for all  $x \in (a, b)$ . Show that  $\frac{f(b)-f(a)}{b-a} > 0$ , and hence that  $f(b) > f(a)$ .

(4) Show that  $|\sin a - \sin b| \leq |a - b|$  for all  $a, b$ .