MATH 100 – WORKSHEET 11
LOGARITHMIC DIFFERENTIATION, APPLICATIONS

1. Logarithmic Differentiation

$$\boxed{(\log x)' = \frac{1}{x}} \qquad \boxed{f' = f \times (\log f)'}$$

(1) Differentiate.

(a) x^x

(b) $(\log x)^{\cos x}$

(c) (Final 2014) Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.

Date: 15/10/2015, Worksheet by Lior Silberman.

2. Applications

Object moves by $s = f(t)$. Then the *velocity* is $v(t) = \frac{\mathrm{d}s}{\mathrm{d}t}$ and the *acceleration* is $a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

(1) The position of a particle at time t is given by $f(t) = \frac{1}{\pi}\sin(\pi t)$.
  (a) Find the velocity at time t, and specifically at $t = 3$.

  (b) When is the particle accelerating? Decelerating?

(2)
  (a) Water is filling a cylindrical container of radius $r = 10$cm. Suppose that at time t the height of the water is $\left(t + t^2\right)$ cm. How fast is the volume growing?

  (b) A rocket is flying in space. The momentum of the rocket is given by the formula $p = mv$, where m is the mass and v is the velocity. At a time where the mass of the rocket is $m = 1000$kg and its velocity is $v = 500\frac{\mathrm{m}}{\mathrm{sec}}$ the rocket is accelerating at the rate $a = 20\frac{\mathrm{m}}{\mathrm{sec}^2}$ and losing mass at the rate $10\frac{\mathrm{kg}}{\mathrm{sec}}$. Find the rate of change of the momentum with time.

(3) A ball is falling from rest in air. Its height at time t is given by

$$h(t) = H_0 - gt_0\left(t + t_0 e^{-t/t_0} - t_0\right)$$

where H_0 is the initial height and t_0 is a constant.
  (a) Find the velocity of the ball. $v(t) =$
  (b) Find the acceleration. $a(t) =$
  (c) Find $\lim_{t\to\infty} v(t)$