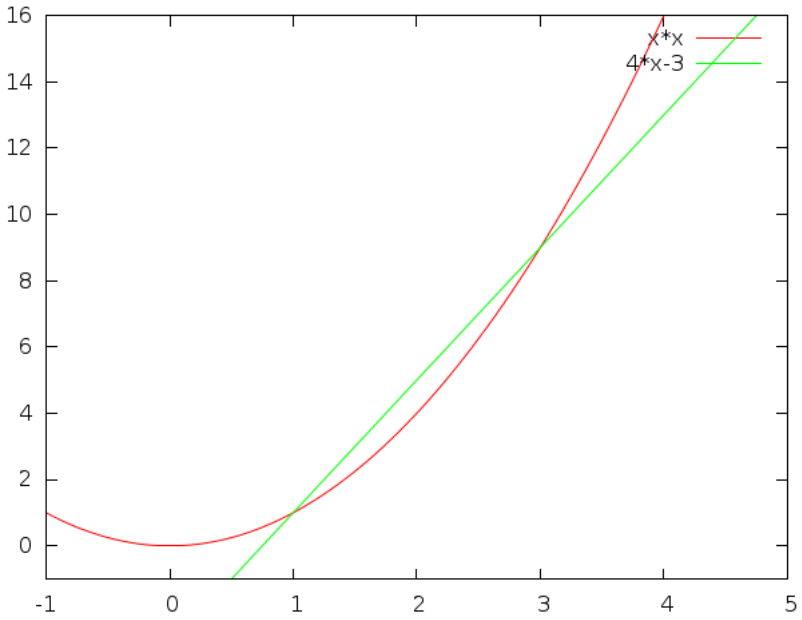
MATH 100 – SOLUTIONS TO WORKSHEET 1
TANGENT AND VELOCITY PROBLEMS

1. The slope of a graph



(1) Find the slope of the line through $P(1,1)$ and $Q(x,x^2)$ where:
   (a) $x=3$
   Solution: $m=\frac{\Delta y}{\Delta x}=\frac{3^2-1}{3-1}=\frac{8}{2}=\boxed{4}$.
   (b) $x=1.1$
   Solution: $m=\frac{1.1^2-1}{1.1-1}=\frac{0.21}{0.1}=\boxed{2.1}$.
   (c) $x=1.01$
   Solution: (using calculator) $m=\frac{1.01^2-1}{1.01-1}=\boxed{2.01}$.
   (d) $x=1.001$
   Solution: (using calculator) $m=\boxed{2.001}$.
   What is the slope of the tangent line at $P(1,1)$? What is its equation?
   Solution: $\boxed{m=2}$ so the line has the equation $y=2(x-1)+1$.

Date: 10/9/2015, Worksheet by Lior Silberman.

2. Limits

(1) Evaluate $f(x) = \frac{x-3}{x^2-x-6}$ at $x = 2.9, 2.99, 2.999, 3.1, 3.01, 3.001$. What is $\lim_{x\to 3} f(x)$?

Solution: (using calculator)

x	2.9	2.99	2.999	3.1	3.01	3.001
$f(x)$	0.204	0.2004	2.0004	1.96	1.996	1.9996

(2) Evaluate

(a) $\lim_{x\to 1} \sin(\pi x)$

Solution: $\boxed{\lim_{x\to 1} \sin(\pi x) = \sin(\pi \cdot 1) = \sin \pi = 0}$.

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

Solution: For $x \neq 1$ we have $\frac{e^x(x-1)}{x^2+x-2} = \frac{e^x(x-1)}{(x-1)(x+2)} = \frac{e^x}{x+2}$ and hence

$$\boxed{\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2} = \lim_{x\to 1} \frac{e^x}{x+2} = \frac{e^1}{1+2} = \frac{e}{3}.}$$

(3) Either evaluate the limit or explain why it does not exist

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

Solution: From the left we have $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x} = \sqrt{1} = 1$, from the right we have $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(2 - x^2\right) = 2 - 1^2 = 1$ so the limit exists and equals $\boxed{1}$.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

Solution: We still have $\lim_{x\to 1^-} f(x) = 1$ but now $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(4 - x^2\right) = 3$ and the limit **does not exist**.