MATH 253 – WORKSHEET 16
OPTIMIZATION

1. CRITICAL POINTS

1.1. **Single-variable.**

Definition 1. $f(x)$ has a *critical point* at x_0 if $f'(x_0) = 0$. If, in addition, $f''(x_0) \neq 0$ call the point *"ordinary"*, and (fact) if $f''(x_0) > 0$ we have a *local minimum*, if $f''(x_0) < 0$ a *local maximum.*

Given $f(x)$ defined on $[a, b]$ we find absolute minimum/maximum by (1) Finding the critical points in (a, b); (2) Evaluating f at every critical point *and at the endpoints* a, b; and (3) Selecting the smallest/largest value seen.

1.2. **Two-variable.**

Definition 2. $f(x, y)$ has a *critical point* at (x_0, y_0) if $\vec{\nabla} f\,(x_0, y_0) = 0$. In that case set $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - f_{xy}^2$ (evaluated at (x_0, y_0)). If $D \neq 0$ call the point *"ordinary"*, and further:

- If $D < 0$ we have a *saddle point*
- If $D > 0$, then $f_{xx} > 0$ at a *local minimum*, $f_{xx} < 0$ at a *local maximum.*

Minimum-finding: given $f(x, y)$ defined on a region R, (1) find the critical points inside R (2) evaluate f on the boundary of R (3) select the smallest/largest value.

2. PROBLEMS

(1) Let $f(x, y) = (2x - x^2)(2y - y^2)$.
   (a) Find and classify the critical points

   (b) Find the absolute maximum and minimum in the domain $R = \{0 \leq x \leq 2,\ 0 \leq y \leq 2\} = [0, 2] \times [0, 2]$.

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(c) Find the absolute maximum and minimum in the domain $R = \{0 \leq x \leq 3,\ 0 \leq y \leq 2\} = [0,3] \times [0,2]$.

(2) Find the equation of the plane which passes through $(1,2,3)$ and encloses the smallest volume in the positive octant.