MATH 253 – WORKSHEET 1
3-SPACE

1. Planes and spheres

(1) Find the point of intersection of the planes with equations $x + y + z = 10$, $x = y$, $z = 5$.

Solution: At a point where $y = x$ and $z = 5$ we have $10 = x + y + z = x + x + 5$ so $2x = 5$ and $x = 5/2$. Since $y = x$ the only solution is $(5/2, 5/2, 5)$.

(2) $x^2 + y^2 + z^2 = 2x + 2y$ is the equation of a sphere. Find its centre and radius.

Solution: Completing the square, the equation is equivalent to $\left(x^2 - 2x + 1\right) + \left(y^2 - 2y + 1\right) + z^2 = 1 + 1$, that is

$$(x-1)^2 + (y-1)^2 + z^2 = \left(\sqrt{2}\right)^2 .$$

The sphere therefore has centre $(1, 1, 0)$ and radius $\sqrt{2}$.

(3) Does the sphere $x^2 + y^2 + z^2 = 2x + 2y$ intersect the plane $z = 5$?

Solution: If $z = 5$ then $(x-1)^2 + (y-1)^2 + z^2 \geq 0 + 0 + 5^2 = 25 > 2$ (every square is non-negative), so no point of the plane lies on the sphere ("the intersection is empty").

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