The University of British Columbia

MATH 253

Midterm 2

13 November 2013

Time: 50 minutes

LAST NAME: ______________________ FIRST NAME: ______________________

STUDENT # : ________________ INSTRUCTOR'S NAME: ______________________

This Examination paper consists of 9 pages (including this one). Make sure you have all 9.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.

MARKING:

Q1		8
Q2		8
Q3		10
Q4		9
TOTAL		35

Names of Instructors: Jim Bryan, Dale Peterson, Sabin Cautis, Lior Silberman

Q1 [8 points]

Consider the integral

$$\iint_T \sqrt{3}\, dA$$

where T is the triangle in the xy-plane with vertices $(0,0)$, $(2,0)$, and $(1,\sqrt{3})$.

(*a*) [2 points] Write the integral as an iterated integral where you integrate x first. **Do not evaluate the integral (yet!).**

(*b*) [2 points] Write the integral as an iterated integral where you integrate y first. Hint: you may write this integral as the sum of two integrals. **Do not evaluate the integral (yet!).**

(*c*) [2 points] Write the integral as an iterated integral in polar coordinates. **Do not evaluate the integral (yet!).**

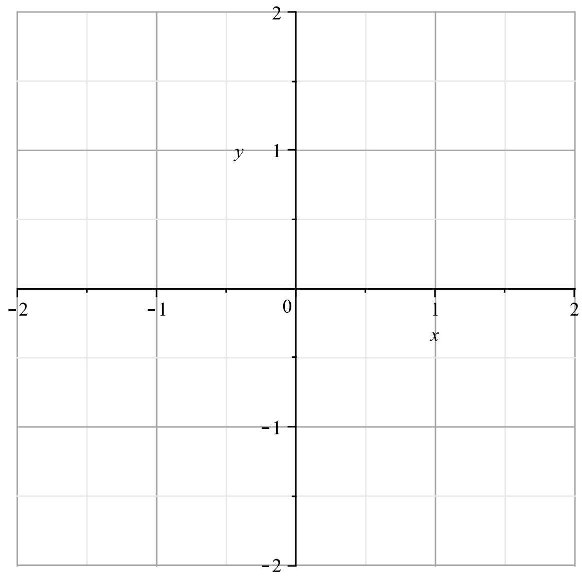
(*d*) [2 points] Evaluate the integral using any method.

Q2 [8 points]

Consider the following iterated integral.

$$\int_{x=0}^{1} \int_{y=1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} x \sin\left(\pi\left(1-y^2+\frac{y^3}{3}\right)\right) dy\, dx$$

(a) [3 points] Sketch the domain of integration on the graph provided below.



(b) [5 points] Compute the integral.

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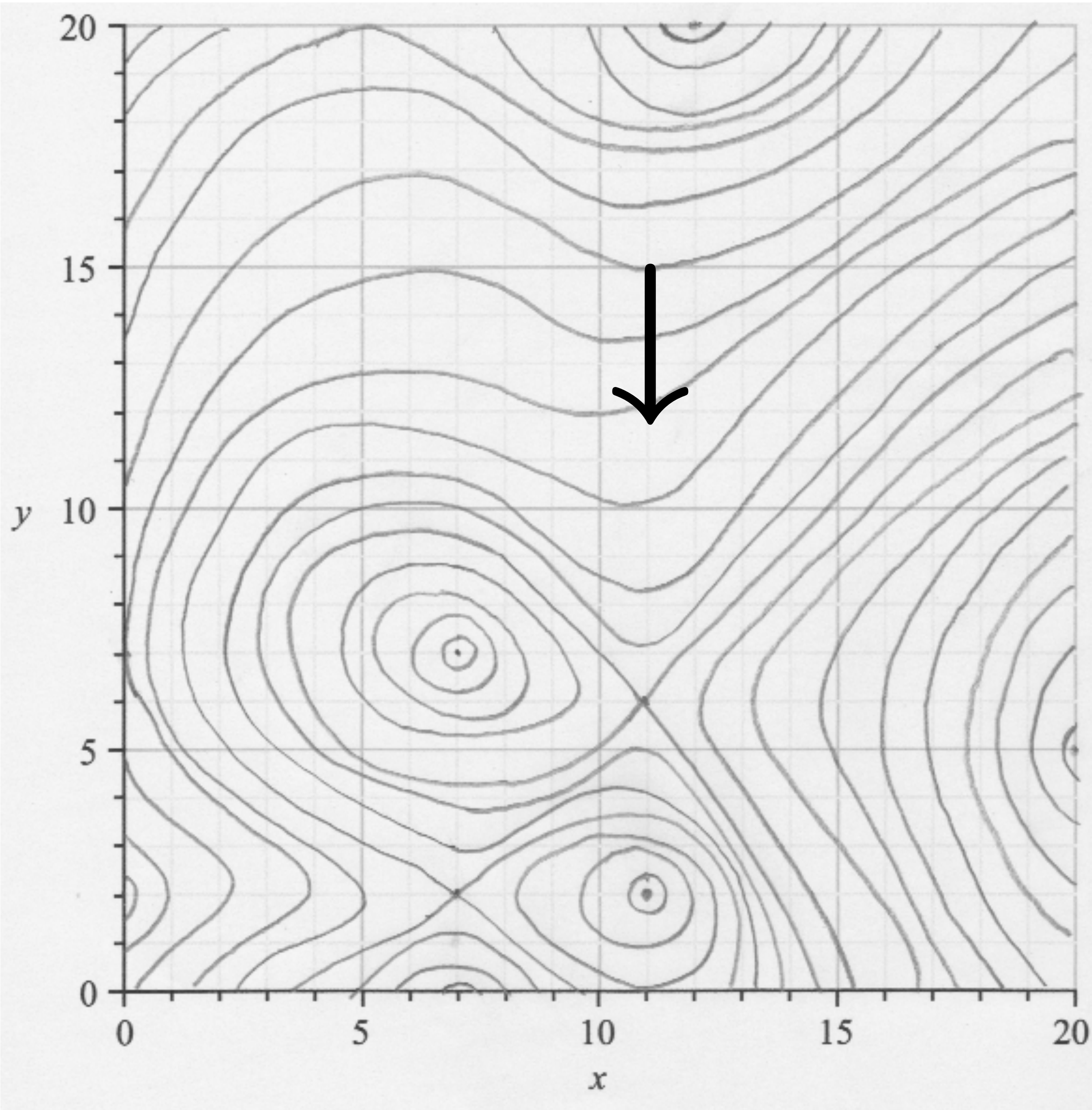
Q3 [10 points]

The following is the contour plot of a function $f(x, y)$ with domain $\{(x, y) : 0 \le x \le 20, 0 \le y \le 20\}$. The values of the contours are spaced evenly. You may make reasonable assumptions about the function: the gradient does not vanish along an entire contour, the function does not fluctuate wildly on a scale smaller that shown by the contours. As indicated below, **The gradient of f at the point (11,15) is given by $\langle 0, -3\rangle$.** Answer the questions on the following page.



(a) (4 points.) Find the coordinates of the critical points of f on the interior of its domain. Classify each critical point as a local maximum, a local minimum, a saddle point, or "other".

(b) (2 points.) Find the coordinates of the global maximum of f and the global minimum of f.

(c) (1 point.) Let $u = \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. Is the value of $(D_u f)(1, 2)$ positive, negative, or zero? Circle the correct answer.

(d) (1 point.) Is the value of $f_{xx}(7, 7)$ positive, negative, or zero? Circle the correct answer.

(e) (1 point.) The direction of the gradient of f at the point $(10, 5)$ is given by which of the following (circle the correct answer):

i) $\frac{1}{\sqrt{2}} \langle 1, 1 \rangle$

ii) $\frac{1}{\sqrt{2}} \langle 1, -1 \rangle$

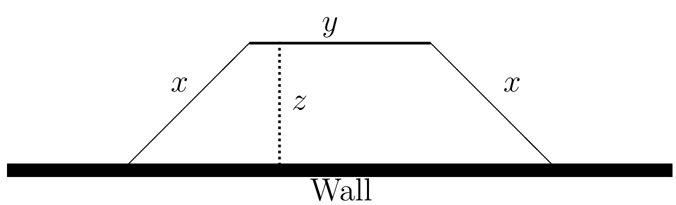
iii) $\frac{1}{\sqrt{2}} \langle -1, 1 \rangle$

iv) $\frac{1}{\sqrt{2}} \langle -1, -1 \rangle$

(f) (1 point.) Let $g(x, y) = 40 - 5y + 2x$. Find the approximate coordinates of the point which maximizes $f(x, y)$ subject to the constraint $g(x, y) = 0$.

Q4 [9 points]

A trapezoidal enclosure is to be constructed by a fence with three sides and an existing wall. Two of the fence sides are the same length x and the third side is length y and is to be parallel to the existing wall. Let z be the distance from the wall to the parallel wall (see the picture). The area of the enclosure is required to be $3\sqrt{3}$ square meters and we wish to determine the fence which uses as little fencing material as possible. Find the values of the side lengths x and y and the distance z for the fence which has the minimal total length of fence. You may assume that $y > 0$ and that $x > z$.



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