# Math 422/501: Groups and Fields
# Fall Term, 2009

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| Main Course Website | `http://www.math.ubc.ca/~lior/teaching/0910/501_F09/` |
| SLATE Website | `https://slate.math.ubc.ca/slate/Slate/2009-2010/Winter_Term1/MATH501` |
| Contact me at | MAT 229B — 604-827-3031 – `lior@math.ubc.ca` |
| My Website | `http://www.math.ubc.ca/~lior/` |
| Class | M 14-16, W 14-15 at MATX 1118[1] |
| Office Hours | By appointment and Mondays 11-12. |
| Textbook | *Various – see below* |
| Course Prerequisites | MATH 322 |

## About the course

This course will cover two basic components of abstract algebra:

- Group theory
- Fields and Galois Theory

Their initial development (most significantly, by Galois) date to the early 19th century and lie at the roots of modern abstract algebra. We will discuss group actions and their implication for the structure theory of groups via the Sylow Theorems, as well as other topics in the structure theory of groups. We will then turn to the theory of fields and their extensions, especially its connection with group theory. This will allow us to solve (negatively) the age-old problems of trisecting the angle, duplicating the cube, and solving polynomial equations by radicals. At the end of the course we will remark on the problem of squaring the circle.

The topics are foundational and covered in many textbooks – the suggestions here are not meant to be exclusive. Basically, any textbook titled "Algebra", "Abstract Algebra" or the like will cover everything. Examples include [1, 3, 4, 5]. Similarly, any textbook titled "Group Theory" such as [7, 8] or "Galois Theory" such as [6, 9] will be sufficient for that part of the course. Clark's textbook [2] is the official "optional" book for the course; it covers all the material we will need at a very reasonable price.

## Teaching and learning

### Learning goals

- Seeing some foundations of abstract algebra.
- Practicing with everyday tools of the trade.
- Applying combinatorial and algebraic machinery (Sylow theorems; the Galois correspondence).

### What you can expect from me

- To come prepared for class: knowing what we want to achieve, and how we will achieve it.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time by e-mail outside class.
- Timely and clear explanations of what is correct in your work and what is not.

### What's expected from you

- Come prepared to class, having read relevant material and done problem sets.
- Actively participate in the course: read ahead of class, think about the material, and ask questions.
- Written work that is readable and communicates your ideas.

## Official Policies

### Learning

- My lectures will assume that you have read some material beforehand. As in any course your main goals are to ***work through examples*** and become ***familiar with the vocabulary and notations***, as well as the ***ideas*** behind proofs. Learning the details of proofs, while useful, is not the point. As explained above, any text will serve for this.
- Assigned problems may be based on prospective reading material, or develop ideas separate from those taught in class.

### Assessment

- Marks will be mostly based on the correctness (rigor) of your proofs, but also on clarity and elegance. Claims which are true but not justified will receive little credit, if any.
- There will be twelve weekly problem sets, to be posted on the course website. They will be due by the end of class a week after they were assigned. I will drop the lowest two scores before calculating the homework grade.
  - Late assignments will not be accepted for credit. In exceptional circumstances (a proof of the emergency and advance notification if possible will be required) a late problem set will be registered (that is, will not be scored a zero) if you finish it and hand it in after the emergency has passed.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently. You may (and should) share your ideas but you may not share your written work.
  - It is possible that only certain problems from a problem set will be selected for grading. Complete solutions will be posted in any case.
- There will be a midterm exam in class on Wednesday, October 7th, as well as a final exam during the usual exam period.
  - If you need special accommodations when taking written exams, please contact the Office of Access & Diversity (`access.diversity@ubc.ca`).

– If the midterm (or final) exam conflicts with a religious observance, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements..

- The final grade will be calculated as follows:

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| Problem sets: 30% |
| Midterm: 20% |
| Final exam: 50% |

## References


[1] Michael Artin. *Algebra*. Prentice Hall Inc., Englewood Cliffs, NJ, 1991.

[2] Allan Clark. *Elements of Abstract Algebra*. Dover Publications, New York, corr. edition, 1984.

[3] David S. Dummit and Richard M. Foote. *Abstract algebra*. John Wiley & Sons Inc., Hoboken, NJ, third edition, 2004.

[4] Nathan Jacobson. *Basic algebra. I*. W. H. Freeman and Company, New York, second edition, 1985.

[5] Serge Lang. *Algebra*, volume 211 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, third edition, 2002.

[6] James Milne. Field and galois theory. Course notes available at `http://www.jmilne.org/math/CourseNotes/math594f.html`.

[7] James Milne. Group theory. Course notes available at `http://www.jmilne.org/math/CourseNotes/math594g.html`.

[8] Joseph J. Rotman. *An introduction to the theory of groups*, volume 148 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, fourth edition, 1995.

[9] Ian Stewart. *Galois Theory*. Chapman & Hall/CRC Mathematics. Chapman & Hall/CRC, Boca Raton, FL, third edition, 2004.

# Tentative Schedule of Lectures & Readings

The following is accurate as of the date on the top of this syllabus; changes will be announced in class and posted to the course website. You should read *ahead* of the relevant classes.

## 1 9/9

General introduction; review of linear algebra and elementary group and ring theory.

## 2 14/9 – 16/9

Group actions; the class formula; $p$-groups.

## 3 21/9 – 23/9

Cauchy and Sylow Theorems; classification of groups of small order.

## 4 28/9 – 30/9

Nilpotent and solvable groups. Commutators. Simple groups.

## 5 5/10 – 7/10

Midterm

## 6 12/10 – 14/10

Fields and field extensions. The ring of polynomials.

## 7 19/10 – 21/10

Degrees of extensions. Trisecting the angle and duplicating the cube. The fundamental theorem of algebra.

## 8 26/10 – 28/10

Normal extensions; splitting fields; normal closure. Separability. Algebraic and separable closures.

## 9 2/11 – 4/11

Field embeddings and automorphisms. Galois groups. The Galois correspondence.

## 10 9/11 – 11/11

Cyclotomic extensions.

## 11 16/11 – 18/11

Solubility of equations by radicals.

## 12 23/11 – 2/12

Transcendence and squaring the circle.

## 13 30/11 – 2/12

Topic to be chosen.